The failure of Mirels' theory to give better results is ascribed to restrictions inherent in his local similarity approach. Only perturbations in u_2 are taken into account, whereas variations in U_s, p_2, and ρ_2 are neglected.

Remaining discrepancies may be explained by other parameters not included in either of the two theories, for example, the diaphragm opening mechanism.

References

[1] Roshko, A. and Smith, J. A., "Measurements of test time in the GALCIT 17-inch shock tube," AIAA J. 2, 186–187 (1964).

[2] Mirels, H., "Shock tube test time limitation due to turbulent-wall boundary layer," AIAA J. 2, 84–93 (1964).

[3] Brocher, E. F., "Hot flow length and testing time in real shock tube flow," Phys. Fluids 7, 347 (1964).

Inadequacy of Nodal Connections in a Stiffness Solution for Plate Bending

Bruce M. R. Irons* and Keith J. Draper†

Rolls-Royce Ltd., Derby, England

IN choosing element displacement functions for a stiffness method of analysis the following criteria must be met:

1) It must be possible to represent the rigid body motions of an element. Otherwise the equilibrium conditions of the element as a whole are falsified.[1]

2) It must be possible to represent states of constant stress. Otherwise, as the mesh of elements is finely subdivided, there is no guarantee that the stresses will converge toward continuous functions; in general, they will not converge at all.[2]

3) Where neighboring elements abut between nodes, there must be no discontinuity of slopes and deflections between the two elements.[3] Otherwise the idealization includes hinges or sawcuts between elements as well as the constraints imposed by the displacement functions. Therefore the bound theorems no longer hold[1, 2, 4] and the solution cannot be described as "pure stiffness."

This note shows that 2 and 3 are incompatible for plate elements in bending, which implies that elements should not be tied at nodes in plate and shell problems but should be matched along boundaries.[5]

Triangle ABC in Fig. 1 is given unit rate of twist about AB, so that at every point $w = xy$. Thus the quantity known as "Torsion" $= \partial^2 w/\partial x \partial y = w_{xy}$, is unity at every point in ABC. It is an easy matter to calculate the nodal rotations θ_1, θ_2, and θ_3 in the directions of the sides, and the nodal deflection δ. At least one of these, applied alone, must give nonzero w_{xy} at A. Say it is δ, θ_2, or θ_3 (or a combination of them) which gives $w_{xy} = K \neq 0$. Consider any undistorted triangle ABC', which is tied to ABC distorted at C. Near A, in ABC

$$\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial x \partial y} \times dx = K\,(dx) \tag{1}$$

whereas in ABC' it is zero. The nonconformity of slope, required to be zero along AB, apparently has a nonzero derivative along AB; this is a contradiction.

Suppose, however, it is θ_1 that gives $w_{xy} = K \neq 0$ at A. Let $AC'B$ be a reflection of ACB about AB, and let both be distorted by θ_1 only. By symmetry, if the deflections on AB are to conform they must be zero, i.e., AB cannot move. Also AC cannot move, otherwise an undistorted triangle ADC could not conform. It follows that, near A, $w = K_1 \times$ (distance from AB) $\times$ (distance from AC) where K_1 is chosen to give the xy term a coefficient K. It follows as before that slope conformity with triangle ADC is impossible.

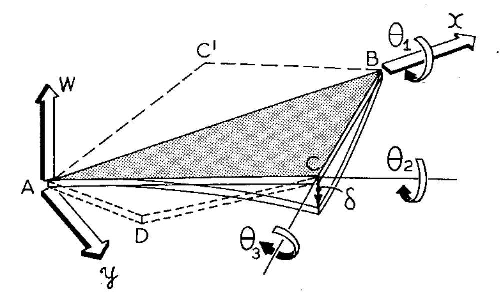

Fig. 1. Illustration of torsional deformation of element ABC with attached triangle.

These arguments extend without difficulty to the polygonal element. If the displacement functions have singularities at the nodes, such that w_{xy} has no unique limiting value, this proof fails.

References

[1] Melosh, R. J., "Basis for derivation of matrices for the direct stiffness method," AIAA J. 1, 1631–1637 (1963).

[2] Irons, B. and Barlow, J., "Comment on 'Matrices for the direct stiffness method,'" AIAA J. 2, 403 (1964).

[3] Pian, T. H. H., "Derivation of element stiffness matrices by assumed stress distributions," AIAA J. 2, 1333–1336 (1964).

[4] Genot, A., "Bornes aux coefficients d'influence," Rev. Universelle Mines 17, 315–326 (May 1961).

[5] Jones, R. E., "A generalization of the direct stiffness method of structural analysis," AIAA J. 2, 821–826 (1964).

Received November 12, 1964.

* Senior Stress Engineer.

† Stress Engineer.

Structural Eigenvalue Problems: Elimination of Unwanted Variables

Bruce Irons*

Rolls Royce Ltd., Derby, England

THE tendency in structural analysis is to use hundreds or thousands of variables, whereas the processes involved in finding eigenvalues of full matrices favor tens of variables, or at most slightly over 100. In frequency calculations the classic (but inefficient) technique is to use discrete masses associated with certain selected deflections. A better technique in simple cases is to use fewer elements and to write the kinetic energy and the strain energy in terms of the same assumed deflected shape, with distributed mass. However, engineers do not usually divide a structure into many elements if it can be avoided.

The proposed method uses distributed mass in the KE but retains only a small proportion of the nodal deflections, hereafter termed "masters." The remaining "slave" deflections take values giving least strain energy, regardless of what this does to the KE. Thus a slave node is assumed free from inertial forces. The argument is most clearly visualized in the case of a cantilever. If one takes a result from a discrete mass calculation, draws a smooth curve through the deflected points, and recalculates the KE from the curve, the natural frequency can be corrected to give tolerably good answers. The practical engineer may use interpolation formulas or french curves, or he may prefer to use a flexible beam to draw his smooth curve. But the best flexible beam to use would be the cantilever itself, especially if it had dis-

Received November 12, 1964.

* Senior Stress Engineer.